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Study of the nature of $\iota/\eta(1440)$ in the chiral perturbation theory approach

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Abstract. The nature of $\iota/\eta(1440)$ is analysed in the framework of the hypothesis that it represents a single pseudoscalar resonance. Assuming that it arises due to the mixing between the glueball and the $q\bar{q}$ nearby states (η, η', and their radial excitations) two upper estimates are obtained for the partial width $\Gamma(\iota/\eta(1440) \to K^*K)$ — one for the case when $\iota/\eta(1440)$ is mainly a glueball and another one when it is mainly a radial excitation of the $s\bar{s}$ state. Both estimates are obtained in the chiral perturbation theory approach taking into account the available data on the vector mesons and the pseudoscalar state $K(1460)$, which is interpreted as a radial excitation of the K meson. The same partial width is independently estimated on the basis of the combined OBELIX and Crystal Barrel data on the production of $\iota/\eta(1440)$ in $p\bar{p}$ annihilation. Comparing the results we show that the glueball content of $\iota/\eta(1440)$ is suppressed while its $s\bar{s}$ radial-excitation interpretation is favoured by the data.

1 Introduction

The pseudoscalar (PS) state $\iota/\eta(1440)$ is traditionally considered as a probable candidate for glueballs. It was first discovered in reaction of $p\bar{p}$ annihilation in the middle of 1960-s [1]. Then it has been studied in a lot of works, both theoretical and experimental [2]. The hypothesis of the glueball origin of $\iota/\eta(1440)$ is based, mainly, on the fact that it is copiously produced in gluon-rich reactions, such as the J/ψ radiative decays and the $p\bar{p}$ annihilation. In addition, the glueball origin of $\iota/\eta(1440)$ is corroborated by the fact that it is seen in various decay modes allowed by strong interactions but is almost invisible in $\gamma\gamma$ collisions.

However the glueball origin of $\iota/\eta(1440)$ has never been proved, since the above arguments remain rather qualitative. Moreover, in the last years there arose a serious doubt that $\iota/\eta(1440)$ is really a glueball. The doubt is caused mainly by the results of lattice calculations [3] which predict the lowest PS glueball with appreciably higher mass than the observed mass of $\iota/\eta(1440)$. Simultaneously, the experimental situation changed because there appeared some new data which indicated that there might be two overlapping PS resonances in the $\iota/\eta(1440)$ region. (This question, however, is not quite clear yet [2].) If the latter result will be confirmed then the $\iota/\eta(1440)$ problem will become more intricate.

A fresh view on the problem of $\iota/\eta(1440)$ has been recently proposed in [4]. In this work $\iota/\eta(1440)$ is considered as a single PS resonance whose nature is attributed to the $s\bar{s}$ radial excitation. Allowing its mixing with the higher mass PS glueball, [4] describes the Mark III data on the production of $\iota/\eta(1440)$ in the J/ψ radiative decays. According to [4], a discrepancy with the experimental works which prefer the two-resonance structure of $\iota/\eta(1440)$ might be due to their use of not quite correct form of the relativistic Breit-Wigner amplitude (the energy-dependence of the decay widths of resonances and of some factors was not taken into account, cf. [5]).

The most significant result of [4] is, apparently, that it suggests a way to eliminate the discrepancy between the experimental results which indicate the glueball origin of $\iota/\eta(1440)$, and the results of the lattice calculations which predict higher masses for the PS glueball. However, the hypothesis of the $s\bar{s}$ origin of $\iota/\eta(1440)$ needs to be further confirmed since not all available data on $\iota/\eta(1440)$ have been taken into consideration in the framework of this hypothesis. Moreover, the very discrepancy mentioned above may turn out to be nonexistent, since the modern lattice calculations [3] in reality are not model-independent as far as the 0^{-+} glueball is concerned.[1] So, the glueball origin of $\iota/\eta(1440)$ cannot be completely excluded.

[1] The point is that there are two different operators of the 0^{-+} glueball in the lattice approach. One of them is defined as a set of three-dimensional loops deformed in some special way (in order to produce the 0^{-+} quantum numbers) [3]. Another operator is a strictly four-dimensional object, since it is defined as the lattice analog of the continuous operator $G_{\mu\nu}\widetilde{G}^{\mu\nu}$ [6]. (The structure of the first one is **BBB** while the structure of the second one is **EB**.) Therefore, they can generate quite different glueball states. Reference [3] used only the first of these two operators. One can suppose that it generates the heavier state (which is presumably the pseudoscalar excitation over the ground-state scalar glueball state) whereas the second operator generates the lighter state

In the present work we carry out further investigation of $\iota/\eta(1440)$ under the assumption of its one-resonance structure. However, in contrast to [4], we use another set of data, namely, the data of OBELIX and Crystal Barrel at LEAR on the $p\bar{p}$ annihilation at rest. Moreover, we consider both the hypotheses on the origin of $\iota/\eta(1440)$ — the one according to which $\iota/\eta(1440)$ is mainly a glueball, and another one according to which it is mainly a radial excitation of the $s\bar{s}$ state. Then both these hypotheses are to be compared in the framework of the same approach. An intermediate case, when $\iota/\eta(1440)$ involves comparable contributions of the glueball and the excited $s\bar{s}$ state, can hardly take place, since then the mixing partner of $\iota/\eta(1440)$ should be visible in the gluon-rich reactions, but it is not the case if $\iota/\eta(1440)$ is a single resonance.

The main idea of the present study is to compare the theoretical estimate of the partial width $\Gamma(\iota \to K^*K)$ with its experimental value. (We designate further ι/η (1440) by a single symbol ι.) The choice of the decay $\iota \to K^*K$ is caused by a possibility of its description. Indeed, since this decay occurs near the threshold its final states have small kinetic energies (in the rest frame of ι). So, the decay $\iota \to K^*K$ may be described in the framework of the chiral perturbation theory (χPT), which is a model-independent method. The experimental estimate of $\Gamma(\iota \to K^*K)$ may be obtained with great accuracy as well, since from the LEAR data it may be obtained without taking into account the contributions of the $\rho\rho$ and $\omega\omega$ channels (which are little-known) to the production of ι. (See Sect. 6 for a detailed discussion of this point.)

The structure of the present work is as follows. In the next Section we propose a chiral effective Lagrangian which describes PS $q\bar{q}$ resonances and the PS glueball. In Sect. 3 the vector mesons are added and the vertices of the decays of PS states to K^*K are discussed. Section 4 shows that the chiral loops do not change the results obtained in the previous sections. In Sect. 5 we obtain the upper bound of $\Gamma(\iota \to K^*K)$ while taking into account the mixing of the PS states and the effect of the finite width of the K^* meson. In Sect. 6 the experimental value of $\Gamma(\iota \to K^*K)$ is estimated from the combined data of OBELIX [7,8] and Crystal Barrel [9]. Section 7 discusses the results. Appendix A collects the formulae which permit to calculate the correction factors caused by the finite widths of intermediate resonances. Appendix B estimates the contribution of the decay $\iota \to \rho\rho$ to the annihilation $p\bar{p} \to \pi\pi\iota$ at rest. Appendix C discusses the amplitude of the $p\bar{p} \to \pi\pi\iota$ in χPT.

2 Excited $q\bar{q}$ states and PS glueball in χPT

In order to define χPT the approach of chiral effective Lagrangian is conventionally employed [10,11]. The fundamental ingredients of this approach are the interpolating fields of observable states involved in the process to be described. The range of application of χPT is bounded by the condition of low momenta of the initial and final states in the center-of-mass frame (usually each three-momentum is required to be much less than the ρ meson mass).

Independently of the kind of the process the octet of the lightest PS states (π, K, η) must be represented in the chiral effective Lagrangian. Since their Goldstone nature the interpolating fields of these states may be collected in a unitary unimodular matrix $u(\phi)$ which takes values in the coset space $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R/\mathrm{SU}(3)_V$ [12]. Here $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ is the Lagrangian chiral-group symmetry and $\mathrm{SU}(3)_V$ is the symmetry of the vacuum in QCD. Under the chiral group $u(\phi)$ transforms non-linearly,

$$u(\phi) \to g_L\, u(\phi)\, h^\dagger \;=\; h\, u(\phi)\, g_R^\dagger, \tag{1}$$

with $g_{L,R} \in \mathrm{SU}(3)_{L,R}$, and $h = h(g_L, g_R, \phi)$ is the compensating $\mathrm{SU}(3)_V$ transformation. In case of the diagonal transformations, $g_L = g_R = g_V$, h equals g_V and, so, h becomes independent on ϕ. Usually $u(\phi)$ is considered in the exponential parameterization,

$$u(\phi) = \exp\{i\phi/F\}, \quad \phi = \sum_{a=1,\ldots,8} \phi^a \lambda^a/2, \tag{2}$$

with ϕ^a and λ^a are the interpolating fields and the Gell-Mann matrices, F is the universal octet decay constant.

The singlet member of the nonet of the lowest PS states (η') must be described as a heavy state since it is not a Goldstone boson. Being singlet its interpolating field, ϕ^0, is invariant under the chiral group. However, ϕ^0 is not invariant under the chiral $\mathrm{U}(1)_A$ rotation [13]:

$$\phi^0 \to \phi^0 + F_0 \omega_5^0. \tag{3}$$

Here ω_5^0 is the parameter of $\mathrm{U}(1)_A$, F_0 is a dimensional constant. The nature of transformation (3) is considered in detail in [14]. Here we notice only that $\mathrm{U}(1)_A$ transformation is the exceptional property of ϕ^0 because all other interpolating fields are invariant under $\mathrm{U}(1)_A$.

Other heavy interpolating fields, if they are not singlets, are not invariant under the chiral group. For instance, the octet heavy states transform like as follows [12,15]:

$$R \to h\, R\, h^\dagger, \quad R = \sum_{a=1,\ldots,8} R^a \lambda^a/2. \tag{4}$$

Here h is the same as in (1). The singlet members of the nonets (R^0) and other SU(3)-singlets (glueballs, for instance) are invariant under the chiral group. So, the chiral symmetry is not sufficient to distinguish between different singlet states, and additional ideas are needed to do that. To that end we shall follow the ideas of [14] (see below).

Excluding the singlet-state problem, the transformation properties of the interpolating fields determine the structure of the chiral effective Lagrangian. In the framework of χPT the Lagrangian is represented in form of the expansion in the derivatives of fields and the current quark masses. The terms without the derivatives are responsible for the mass spectrum of the observable states. In case when the effective theory is to describe the ground-state

PS mesons (ϕ^0, ϕ^a), their radial excitations (P^0, P^a), and the ground-state PS glueball (G), the mass terms at order $p^0 + p^2$ are determined by the following chiral-invariant Lagrangian:

$$L^{\rm mass} = -\tfrac{1}{2}A_0(P^0)^2 - A\langle P^2\rangle$$
$$-\tfrac{1}{2}M_0^2(\phi^0)^2 - \tfrac{1}{2}M_G^2G^2 - q\,\phi^0 G - \tilde{\alpha}_0(\tfrac{\lambda^0}{2})^2\langle P^0P^0\chi_+\rangle$$
$$-\alpha_0\tfrac{\lambda^0}{2}\langle P^0P\chi_+\rangle - \alpha\langle PP\chi_+\rangle + i\tfrac{F}{2}\beta_0\tfrac{\lambda^0}{2}\langle P^0\chi_-\rangle$$
$$+i\tfrac{F}{2}\beta\langle P\chi_-\rangle + i\tfrac{F}{2}\tilde{\beta}_0\tfrac{\lambda^0}{2}\langle\phi^0\chi_-\rangle + \tilde{\gamma}_0(\tfrac{\lambda^0}{2})^2\langle\phi^0\phi^0\chi_+\rangle$$
$$+\gamma_0(\tfrac{\lambda^0}{2})^2\langle\phi^0P^0\chi_+\rangle + \gamma\tfrac{\lambda^0}{2}\langle\phi^0P\chi_+\rangle + \tfrac{F^2}{4}\langle\chi_+\rangle. \quad (5)$$

(Notice, the particular expression (5) for the Lagrangian is not $\mathrm{U}(1)_A$ invariant, but it may be made invariant by means of replacing ϕ^0 to the $\mathrm{U}(1)_A$-invariant combination $\phi^0 + F_0\Theta$ with Θ is a source of the gluon anomaly operator in QCD [11,13].) In formula (5) the brackets $\langle\cdots\rangle$ mean the trace operation, $\lambda^0 = \sqrt{2/3}$. Parameters A_0, A, M_0, and M_G describe the masses of P^0, P^a, ϕ^0, and G, respectively. Parameter q describes $\phi^0 - G$ mixing in the chiral limit (the limit of the massless quarks and the switched-off mass-like external field). The linear in the current quark masses contributions are described by parameters α, β, γ's and by quantities $\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u$ [11,15]. Here $\chi = 2BM$ with B is proportional to the condensate of quarks, and M is a mass-like external field. Simultaneously M describes the contributions of the current quark masses; when the external field is switched off, $M = \mathrm{diag}(m_u, m_d, m_s)$. With the switched-on external field $\chi_\pm$ transform like P, providing thus the Lagrangian with the chiral invariance. With the switched-off external field $\chi_\pm$ describe the chiral symmetry breaking. In addition, with $m_s \neq m_{u,d}$, $\chi_\pm$ describe the flavour symmetry breaking.

Now let us discuss the singlet field contributions. Note, P^0 and G are involved not symmetrically in (5). This is caused by the theorem [14] which states that any heavy singlet PS interpolating field, which is different from ϕ^0, may not contribute both to terms which involve $\chi_\pm$ and to the term which describes the mixing of this state with ϕ^0 in the chiral limit. Due to this theorem there are two alternative ways to involve a heavy singlet PS state to the effective theory.

In case of the glueball state we use the possibility according to which the chiral-limit mixing between G and ϕ^0 is allowed but contributions of both G and $\chi_\pm$ to the same terms are suppressed. This choice is caused by the following reasons. First, the $\phi^0 - G$ mixing should indeed take place, so long as QCD is possessed of the annihilation mechanism which permits transition between $q\bar{q}$ singlet states and gluonic colorless states. Second, in QCD the quark-gluon interaction does not distinguish the quark flavours. So, the interpolating field of the genuine glueball should not contribute to terms which break down the flavour symmetry. Consequently G should not contribute to terms which involve $\chi_\pm$. Let us note, that we could expect the latter property to be valid not only in the next-to-leading order but rather in the all orders of χPT. (About the possibility to introduce the genuine-glueball interpolating field, especially with taking into account the UV renormalization in QCD, see [14].)

In case of the excited state P^0 we use another possibility according to which the $\phi^0 - P^0$ mixing is suppressed in the chiral limit but instead the contribution of P^0 to terms which involve $\chi_\pm$ is allowed. This choice is caused by the result of the reverse assumption. Indeed, let us assume that there is the $\phi^0 - P^0$ mixing in the chiral limit. Then the excited state P^0 can transform to a non-excited state (ϕ^0) without emission of strong-interacting massless particles — the pions and kaons in the chiral limit. However, if there is not mass (energy) gap then such particles should necessarily be emitted in course of any transformation of the excited state. So, the absence of the emission contradicts to the condition that P^0 is the excited state. Therefore, the above assumption is wrong. Notice, analogously one can show that the mixing between P^0 and G is suppressed as well. The same result follows also from the consistency condition: after G was integrated out the $P^0 - \phi^0$ mixing in the chiral limit would not appear if there was not the $P^0 - G$ mixing.

Extracting from (5) the quadratic terms we can describe the spectrum of the observable states. In the channel of pions and kaons we obtain

$$L^{\rm mass}_{(\pi,K)} = -\tfrac{1}{2}M_\pi^2(PP)^\pi - \tfrac{1}{2}M_K^2(PP)^K$$
$$+\beta\left[m_\pi^2(P\phi)^\pi + m_K^2(P\phi)^K\right]$$
$$-\tfrac{1}{2}m_\pi^2(\phi\phi)^\pi - \tfrac{1}{2}m_K^2(\phi\phi)^K. \quad (6)$$

Here m_π^2 and m_K^2 are the masses of the pions and kaons, $M_\pi^2 = A + 2\alpha m_\pi^2$ and $M_K^2 = A + 2\alpha m_K^2$ are the masses of the excited states P^π and P^K. (The mixings $\phi^\pi - P^\pi$ and $\phi^K - P^K$, which are controlled by β, give rise to the corrections to the masses of order p^4. As far as such corrections are beyond the level of accuracy of (6), we neglect these mixings.) Identifying P^π and P^K with the real states $\pi(1300)$ and $K(1460)$ [2,16] one can estimate the relevant parameters of the Lagrangian: $A = (1.28\ \mathrm{GeV})^2$, $\alpha = 0.49$.

In the isosinglet channel we obtain (with taking into account $\tilde{\beta}_0 = 1$ [14], and assuming, for simplicity, $\tilde{\alpha}_0 = (\alpha_0 + \alpha)/2$)

$$L^{\rm mass}_{(0,8,G)} = -\tfrac{1}{2}M_N^2(P^NP^N) - \tfrac{1}{2}M_S^2(P^SP^S)$$
$$-M_{NS}^2(P^NP^S) + m_\pi^2(P^N\widetilde{\phi}^N) + (2m_K^2 - m_\pi^2)(P^S\widetilde{\phi}^S)$$
$$-\tfrac{1}{2}M_G^2G^2 - q\,\phi^0G - \tfrac{1}{2}\Big(M_0^2 - 2\tilde{\gamma}_0\tfrac{2m_K^2+m_\pi^2}{3}\Big)(\phi^0)^2$$
$$-\tfrac{1}{2}\tfrac{4m_K^2-m_\pi^2}{3}(\phi^8)^2 - 2\sqrt{2}\tfrac{m_\pi^2-m_K^2}{3}(\phi^0\phi^8). \quad (7)$$

Here $M_N^2 = \frac{1}{3}[2A_0 + A + (2\alpha_0 + \alpha)2m_\pi^2]$ and $M_S^2 = \frac{1}{3}[A_0 + 2A + (\alpha_0 + 2\alpha)(4m_K^2 - 2m_\pi^2)]$ are the masses of the excited states $P^N = \sqrt{2/3}P^0 + \sqrt{1/3}P^8$ and $P^S = \sqrt{1/3}P^0 - \sqrt{2/3}P^8$. Parameter $M_{NS}^2 = (\sqrt{2}/3)[A_0 - A + (\alpha_0 - \alpha)2m_K^2]$ describes their mutual mixing. The interpolating fields $\widetilde{\phi}^N$ and $\widetilde{\phi}^S$ involve parameters β's and γ's.

As far as these parameters describe the next-to-leading order of Lagrangian (5), one may neglect the differences between these parameters because these differences are additionally suppressed in the large-N_c expansion. So, let us put $\beta_0 = \beta$ and $\gamma_0 = \gamma$. In this approximation $\widetilde{\phi}^N = \gamma\sqrt{2/3}\phi^0 + \beta\sqrt{1/3}\phi^8$, $\widetilde{\phi}^S = \gamma\sqrt{1/3}\phi^0 - \beta\sqrt{2/3}\phi^8$.

In the isosinglet channel it is too difficult to estimate the parameters because of their multiplicity. Nevertheless, the problem may be simplified if one identifies P^N with $\eta(1295)$ which has been only seen in the $\eta\pi\pi$ channel [2, 17]. This identification is corroborated by the phenomenological equality $M_N^2 \approx M_\pi^2$ and by observation that $A_0 = A$ and $\alpha_0 = \alpha$ in the limit of the large N_c. The direct consequence of this identification is the ideal mutual mixing between the isosinglet PS excited states ($M_{NS}^2 = 0$). In this approximation $M_S^2 = 2M_K^2 - M_\pi^2 \simeq (1.6\,\mathrm{GeV})^2$.

3 Vector mesons in χPT

Now let us involve the vector mesons $(\rho, \omega, \varphi, K^*)$ and discuss their interactions with the PS mesons. Namely, we shall be interested in the vertices of the kind $V\phi\phi$, $VP\phi$ and $VG\phi$. To the purpose there will be needful the following auxiliary quantities composed on the interpolating fields of the light PS mesons:

$$\Gamma_\mu = \frac{1}{2}\left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right),\ u_\mu = \frac{i}{2}\left(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right). \quad (8)$$

Since Γ_μ transforms inhomogeneously under the chiral group it allows one to define the covariant derivative of the heavy fields:

$$\nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R]. \quad (9)$$

Quantity u_μ transforms homogeneously, so it is simply a vector-like building block.

The leading-order chiral effective Lagrangian which describes $V\phi\phi$ interaction, and which is chiral invariant, p- and c-parity even, is as follows [18]:

$$L_{V\phi\phi} = -ig\langle V_{\mu\nu}[u_\mu, u_\nu]\rangle - ig'\langle V_\mu[u_\mu, \chi_-]\rangle. \quad (10)$$

Here V_μ is the vector-meson interpolating field, $V_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu$ is the tensor of the vector field. Notice, in spite of the "naive" chiral counting rules which require the chiral dimension of Lagrangian (10) to be 3, the true leading term of the Lagrangian is of order p^1. Indeed, the first term in (10) may be represented in form of the expansion

$$-ig\langle V_{\mu\nu}[u_\mu, u_\nu]\rangle = -ig/F^2\langle V_{\mu\nu}[\partial_\mu\phi, \partial_\nu\phi]\rangle + \cdots. \quad (11)$$

Here the ellipsis means multi-ϕ contributions. Transferring one derivative from $[\partial_\mu\phi, \partial_\nu\phi]$ to $V_{\mu\nu}$ and taking into account the equation of motion $\partial_\mu V_{\mu\nu} = -M_V^2 V_\nu + \cdots$, one can reduce the number of derivatives in (11). As a result the true leading term of Lagrangian (10) is

$$L_{V\phi\phi}^{(\mathrm{vertex})} = -2ig_{V\phi\phi}\langle V_\mu[\phi, \partial_\mu\phi]\rangle. \quad (12)$$

Here $g_{V\phi\phi}$ means the low-energy coupling constant. The chiral corrections to (12) begin with order p^3.

The chiral-invariant, p- and c-parity even Lagrangian which is responsible for $VP\phi$ interaction at order $p^1 + p^3$ is as follows:

$$\begin{aligned} L_{P\phi\phi} &= -ig_1\langle V_\mu[P, u_\mu]\rangle - ig_2\langle V_{\mu\nu}[\nabla_\mu P, u_\nu]\rangle \\ &- ig_3\langle V_\mu[\nabla_\mu P, \chi_-]\rangle - ig_4\langle V_\mu\{\chi_+, [P, u_\mu]\}\rangle \\ &- ig_5\langle V_\mu\{u_\mu, [P, \chi_+]\}\rangle - ig_6\langle V_\mu\{P, [\chi_+, u_\mu]\}\rangle. \end{aligned} \quad (13)$$

Here under P we understand the sum of the octet fields P^a multiplied by $\lambda^a/2$ and the singlet fields P^0 and ϕ^0 (the singlet fields may contribute with their own coupling constants). Let us notice, that in (13) only the first term of the expansion $\chi_+ = 4BM + \cdots$ is relevant, since all other terms of the expansion are responsible for the higher vertices which involve too many pseudoscalar fields (so χ_+ plays the role of a "spurion"). The leading-order $VP\phi$ vertex implied by (13) is as follows

$$L_{VP\phi}^{(\mathrm{vertex})} = -2ig_{VP\phi}\langle V_\mu[P, \partial_\mu\phi]\rangle. \quad (14)$$

It has the chiral dimension 1 and the chiral corrections beginning with order p^3, as well as in (12).

Let us note, that the singlet fields do not contribute to (14) since the vanishing commutator. Actually, this property is manifestation of the well-known selection rule [19] imposed by SU(3) symmetry for c-parity even singlet PS state decays. However, with the symmetry is broken this selection rule is no longer valid. Indeed, due to the last term in (13) the decay $P^0 \to V\phi$ is possible owing to the "spurion" χ_+.

The above results may be generalized for the PS glueball, as well, but one has to remember that the genuine glueball must not contribute to the $\chi_\pm$-involving terms. The corrections caused by the higher derivatives must be suppressed, too, because of the above selection rule [19]. So, in case of the glueball we have in any order of χPT

$$L_{VG\phi}^{(\mathrm{vertex})} = 0. \quad (15)$$

4 Loop corrections

Strictly speaking, the above analysis may not be complete until the chiral loop corrections are taken into consideration. It is well-known that in case when only the light PS mesons are involved the chiral loops contribute to order p^{d+2} if they are calculated on the basis of order p^d [11]. However, with the heavy fields are involved this picture may change. Let us verify whether this is the case.

To begin with the analysis let us notice that in order to derive only the leading loop corrections one not necessarily has to observe the mixing effects caused by the current quark masses. So, let us neglect these mixings and retain in the Lagrangian only the $\phi^0 - G$ mixing, which is solely the heavy-state mixing. Then we may immediately proceed to the formalism of the heavy static fields with fixed four-velocity v_μ, $v^2 = 1$ [20, 21]. In this formalism virtual heavy

states cannot be destroyed or created, but can transform to other heavy states with almost the same four-velocity v'_μ, with $v'_\mu - v_\mu = O(p)$. A transition to this formalism is provided by the formula $R(v;x) = \sqrt{2M}\exp\{iMvx\}R(x)$ where M is a typical mass of the heavy states,[2] and $R(v;x)$ is a low-frequency field that depends on the four-velocity. Since four-momenta of the heavy states are of the form $P = Mv+k$, with $k = O(p)$, one gets $P^2-M^2 = 2M(kv)+O(p^2)$. So, the dependence on the large mass M disappears in the propagators of the heavy states, but appears instead in denominators of the heavy state vertices.

Basing on this result one may estimate the order in the chiral expansion of any chiral-loop diagram. In particular, the chiral dimension D of a diagram with one heavy-field line going through the diagram is [22]

$$D = 2L+1+\sum_{d=2,4,\ldots}(d-2)N_d^{(l)}+\sum_{d=1,2,\ldots}(d-1)N_d^{(hl)}. \quad (16)$$

Here L denotes the number of light-meson loops, $N_d^{(l)}$ ($N_d^{(hl)}$) counts the number of light-meson (heavy-and-light-meson) vertices of the chiral dimension d. Notice that $D \geq 2L+1$, owing to (16). As applied to the vertices of the previous section this result means that the chiral loop corrections begin with order p^3. Let us emphasize that this is the same order in which the usual chiral corrections begin with to the vertex $VP\phi$.

In case of the glueball-involving vertex $VG\phi$ one has to carry out more detailed analysis which would take into account the property that the current quark masses should not contribute to the vertices which describe the glueball interactions. Let us recall that this condition is the external one with respect to the effective theory. So, it must be satisfied in the presence of the chiral loops as well as in their absence. It is clear that in the presence of the chiral loops it may be only satisfied when there are not chiral loop corrections to the glueball-involving vertices. Really, the chiral loops always produce quark-mass-dependent factors like $m_q \ln m_q$ (remember, the heavy static fields do not form closed loops). So, since the quark-mass dependence is suppressed in the glueball-involving vertices, the chiral loops must not contribute to them. The mechanism ensuring this effect consists in the property that the vertices which involve both G and the light PS mesons are suppressed in the effective theory. Indeed, if in the chiral effective Lagrangian there are bare vertices $RG(\phi)^n$, $n \geq 1$, with R is a heavy static field, then via the tadpole diagrams these vertices give rise to the quark-mass-dependent factors in the renormalized vertices $RG(\phi)^{n-2}$. (In case with $n = 1$ one might consider more complicated diagrams which involve more than one bare vertex.) Thus, so long as the quark-mass dependence is suppressed in the vertices $RG(\phi)^{n-2}$, the Lagrangian vertices $RG(\phi)^n$ must be suppressed from the very beginning. As a result, the full renormalized vertices $RG(\phi)^n$ are suppressed as well.[3]

Now let us discuss the chiral loop corrections to the Lagrangian that describes the spectrum of the heavy PS states. In accordance with (16) these corrections begin with order p^3. However the more detailed analysis shows that the relevant diagrams are the one-loop ones of the type of the self-energy with two $VP\phi$ vertices. In the leading order such diagrams contribute only to the kinetic terms of the heavy states, which in the static-field formalism are of the form $iR^\dagger(kv)R$. So, the p^3-order loop corrections manifest themselves as $[1 + O(p^2)]$-renormalization of the heavy-state wave functions. This effect may not change the results of Sect. 2. It is clear, also, that it does not change the above results about the corrections to the vertices. The corrections that arise from the wave-function renormalization of the light PS mesons and the vector mesons are of the same property.

So, the above discussion shows that the chiral loops do not change our results obtained in the quasi-classical (loop-free) approximation. In particular, there are not chiral corrections to the vertex $VG\phi$ which is zero. The corrections to the vertex $VP\phi$ arise at order p^3 which is higher by two units as compared with the leading order p^1 of this vertex. In accordance with the current practice such corrections may be estimated as 20% of the leading-order result. (It should be noted, that the individual one-loop corrections, that arise from the vector-meson wave function renormalization, are relatively large [21]. Nevertheless, their flavour-non-symmetric parts are small. So, one can redefine χPT attributing the common large flavour-symmetric corrections to the leading-order result — i.e. to the flavour-symmetric coupling constants, etc. — and the remaining small parts of the corrections to the proper corrections. Thus the above statement about the 20%-estimate of the chiral corrections remains in force.)

5 $\Gamma(\iota \to K^*K)$ in χPT

Assuming that ι arises due to the mixing of the pure glueball, isoscalar lowest $q\bar{q}$ states, and their radial excitations, let us present the interpolating field of ι in form of the following decomposition

$$P^\iota = \mathcal{O}_8^\iota\phi^8 + \mathcal{O}_0^\iota\phi^0 + \mathcal{O}_S^\iota P^S + \mathcal{O}_N^\iota P^N + \mathcal{O}_G^\iota P^G. \quad (17)$$

[2] We suppose that the mass splitting of the heavy states is numerically of order $O(p)$ or less. In cases of our interest this property takes place. Really, the mass splitting between ι and K^* is of order $O(p)$, since $M_\iota \approx M_{K^*}+m_K$. The mass splitting among the PS excited states and among the vector mesons is of order $O(p^2)$ in both cases — see (6), (7), and [21]. Notice, one may neglect such mass splittings deriving the leading loop corrections

[3] Let us note, that the above discussion concerns one-particle-irreducible diagrams only. Concerning one-particle-reducible diagrams, they may well describe an interaction between the glueball and the light PS mesons, but only through the $G-\phi^0$ mixing in the external lines outgoing from the vertices. Moreover, such diagrams may yield an effective quark-mass dependence in the glueball interactions with $q\bar{q}$ states, but only through the external-line mixing which occurs outside the vertices

Here $\mathcal{O}_j^n$ is the orthogonal mixing matrix defined on the basis of Lagrangian (7). Further we assume $\mathcal{O}_N^\iota = 0$, thinking that P^N is identical with $\eta(1295)$. Due to (12), (14), (15), and (17) the amplitude of the decay $\iota \to K^*K$ is as follows

$$\mathrm{Amp}\,(\iota \to K^*K) = \left(\sqrt{3}\, g_{V\phi\phi}\mathcal{O}_8^\iota - \frac{1}{\sqrt{2}}\, g_{VP\phi}\mathcal{O}_S^\iota\right)\epsilon_\mu p_\mu. \tag{18}$$

Here ϵ_μ and p_μ are the polarization vector and the four-momentum of the K^* and K. Taking into account factor 4 caused by the presence of two neutral and two charge modes in the K^*K system, and taking into account the equality $\sum_n \epsilon_\mu^{(n)}\epsilon_\nu^{(n)} p_\mu p_\nu = |\mathbf{p}|^2 M_\iota^2/M_{K^*}^2$ where $\mathbf{p}$ is the kaon momentum in the ι rest frame, we obtain the partial width of the decay:

$$\Gamma(\iota \to K^*K) = \frac{1}{2\pi}\left(\sqrt{3}\, g_{V\phi\phi}\mathcal{O}_8^\iota - \frac{1}{\sqrt{2}}\, g_{VP\phi}\mathcal{O}_S^\iota\right)^2 \frac{\xi\,|\mathbf{p}|^3}{M_{K^*}^2}. \tag{19}$$

Here ξ is a correction factor caused by the resonance properties of the K^* meson (see Appendix 1). With M_ι approaches the threshold $(M_{K^*}+m_K)$, ξ grows rapidly, thus compensating partly the decrease of the phase volume. In distance of the threshold and/or neglecting the width of the K^* meson, ξ approaches 1. With $M_\iota = 1416 \pm 6$ MeV, which is the mean value of the Crystal Barrel and OBELIX data (see below), one has $\xi = 1.56^{-0.21}_{+0.37}$. (For comparison: with $M_\iota = 1440$ MeV, $\xi = 1.07$.)

Coupling constants $g_{V\phi\phi}$ and $g_{VP\phi}$ may be estimated on the basis of the PDG data [2]. Thus, from the vector meson data one can obtain (with the leading χPT corrections are taken into account) [23]

$$g_{V\phi\phi}^2/4\pi \simeq 2.9. \tag{20}$$

Constant $g_{VP\phi}$ may be estimated on the basis of the data $K(1460) \to \rho K$, $K^*\pi$ [2]. With help of (7) and (14) we obtain $g_{VP\phi}^2/4\pi \simeq 1.2$, 2.6, respectively. A noticeable difference in the results may be explained by inaccuracy in the experimental data, and by the fact that the final states are not enough soft in the case of these decays (therefore, the chiral corrections may be large). So, let us make use for the constant $g_{VP\phi}$ the rough upper bound which numerically coincides with (20) and which, we believe, should cover the above uncertainties,

$$g_{VP\phi}^2/4\pi < 2.9. \tag{21}$$

Then we obtain the corresponding upper bound of the width

$$\Gamma(\iota \to K^*K) \;<\; \left(\sqrt{6}\,|\mathcal{O}_8^\iota| + |\mathcal{O}_S^\iota|\right)^2 \frac{2.9\,\xi\,|\mathbf{p}|^3}{M_{K^*}^2}. \tag{22}$$

Further we consider two cases — the first one when ι is mainly a glueball, and the second one when ι is mainly a radial excitation of the $s\bar{s}$ state. In the first case the mass of the excited $s\bar{s}$ state must be in the range or (due to the mixing) somewhat higher than 1.6 GeV. However no PS state has been seen in this mass range in the gluon-rich reactions (in the channels K^*K, $K\bar{K}\pi$). So, the excited $s\bar{s}$ state can only be weakly mixed with the PS glueball. With this property the analysis based on Lagrangian (7) gives estimate [14]

$$|\mathcal{O}_8^\iota| \;\simeq\; \frac{\sqrt{8}}{3}\,\frac{m_K^2 - m_\pi^2}{M_\iota^2}\,|\mathcal{O}_0^\iota| \;\simeq\; 0.1\,|\mathcal{O}_0^\iota|. \tag{23}$$

(Remember, there is not direct $\phi^8 - G$ mixing in the Lagrangian, but this mixing may occur indirectly, via the $\phi^8 - \phi^0$ and $\phi^0 - G$ mixings.)

Now, let us consider the condition of the glueball quality of ι: $|\mathcal{O}_G^\iota| > |\mathcal{O}_j^\iota|$, $j = 8, 0, S$. Together with the trivial probability condition $|\mathcal{O}_G^\iota|^2 + \sum_j |\mathcal{O}_j^\iota|^2 = 1$ it leads to estimate $\sqrt{6}\,|\mathcal{O}_8^\iota| + |\mathcal{O}_S^\iota| < 0.75$. As a result, with $M_\iota = 1416 \pm 6$ MeV we obtain

$$\Gamma(\iota \to K^*K) \;<\; 8.2 \pm 3.5\ \mathrm{MeV}. \tag{24}$$

The error in (24) is determined as the sum (in quadratures) of the statistical error caused by inaccuracy in M_ι and the systematical error caused by χPT uncertainties in (22) and (23), which we estimate to be 20% in the amplitude. Let us note, that using the more strong condition of the glueball quality of ι one may obtain the more strong estimate of the width. For example, with $|\mathcal{O}_G^\iota|^2 > \sum_j |\mathcal{O}_j^\iota|^2$ one obtains

$$\Gamma(\iota \to K^*K) \;<\; 7.7 \pm 3.3\ \mathrm{MeV}. \tag{24$'$}$$

In case when ι is mainly the $s\bar{s}$ excited state, the simplest way to estimate the width is to put $\mathcal{O}_8^\iota = 0$, $|\mathcal{O}_S^\iota| = 1$. (Then $\Gamma(\iota \to K^*K) \simeq 14$ MeV.) However this estimate is rather naive and cannot be realistic since the pure $s\bar{s}$ excited state cannot satisfy the ι-properties. To obtain realistic estimate one must demand [4] noticeable mixing between the $s\bar{s}$ excited state and the higher mass PS glueball. However, in virtue of (7) the $P^S - G$ mixing is only possible via the $P^S - \widetilde{\phi}^S(\phi^0)$ and $\phi^0 - G$ mixings. Therefore, the $P^S - \phi^8$ mixing should be noticeable, too. Moreover, with the group factor $\sqrt{6}$ the contribution of $\mathcal{O}_8^\iota$ in (22) may turn out to be significant. However, in contrast to the previous case, we cannot estimate it. So, let us estimate the maximal upper bound of the width. It follows from the possibly weakest conditions $|\mathcal{O}_S^\iota| > |\mathcal{O}_8^\iota|$, $|\mathcal{O}_S^\iota|^2 + |\mathcal{O}_8^\iota|^2 < 1$, under which we obtain

$$\Gamma(\iota \to K^*K) \;<\; 87\ \mathrm{MeV}. \tag{25}$$

Notice, estimate (25) is saturated when $|\mathcal{O}_8^\iota| = |\mathcal{O}_S^\iota| = 1/\sqrt{2}$. But this condition may not be real. So, the true value of the width is, apparently, not too close to the upper boundary indicated in (25). Unfortunately, we cannot propose more strong estimate.

6 $\Gamma(\iota \to K^*K)$ from LEAR data

The experimental value of $\Gamma(\iota \to K^*K)$ has been presented neither in the PDG [2] nor in the original works.

In principle, one may extract it from the available data on the J/ψ radiative decays (Mark III, DM2) and the $p\bar{p}$ annihilation at rest (LEAR). The more preferable between them are the LEAR data (OBELIX + Crystal Barrel) because these data were collected with greater statistics and, what is more important, they permit to extract $\Gamma(\iota \to K^*K)$ without additional assumptions. Namely, one may extract it little knowing parameters of the (probable) decays $\iota \to \rho\rho$ and $\iota \to \omega\omega$. The point is that so long as these decays occur under the nominal threshold, they may be noticeable only with the large invariant mass of ι. On the other hand, in the $p\bar{p}$ annihilation at rest the creation of ι with large invariant mass is suppressed by the phase volume, which is rapidly decreasing with the invariant mass of ι is increasing. As a result, the contributions of $\rho\rho$ and $\omega\omega$ to the creation of ι in the $p\bar{p}$ annihilation at rest turn out to be negligible (see Appendix 2). This situation is drastically different from that which takes place in the J/ψ radiative decays, where the invariant mass of ι is practically unlimited by the phase volume and, therefore, the $\rho\rho$ and $\omega\omega$ contributions to the creation of ι may turn out to be significant [5].

So, we shall use the OBELIX and Crystal Barrel data only. Remember, OBELIX saw ι in the $KK\pi$ modes produced via the K^*K and in the direct three-particle decays. Crystal Barrel saw ι in the $\eta\pi\pi$ modes. Under the assumption that ι is a single resonance OBELIX presented its results in the framework of two fits [7]. In the first fit there were $M_\iota = 1426 \pm 2$ MeV, $\Gamma_\iota = 78 \pm 4$ MeV, in the second one $M_\iota = 1410 \pm 2$ MeV, $\Gamma_\iota = 56 \pm 6$ MeV. Crystal Barrel [9] obtained $M_\iota = 1409 \pm 3$ MeV, $\Gamma_\iota = 86 \pm 10$ MeV. The statistical mean values [2] of these results are $M_\iota = 1416 \pm 6$ MeV, $\Gamma_\iota = 73 \pm 4$ MeV.

Crystal Barrel [9] presented the absolute branching ratio $B(p\bar{p} \to \pi\pi\iota,\, \iota \to \eta\pi\pi) = (3.3 \pm 1.0) \times 10^{-3}$. This result implies

$$B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to \eta\pi\pi) = (2.2 \pm 0.9) \times 10^{-3}. \quad (26)$$

OBELIX [8] obtained

$$B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to KK\pi) = (1.80 \pm 0.15) \times 10^{-3}. \quad (27)$$

From (26) and (27), neglecting other possible decays of intermediate ι, there follows

$$B(p\bar{p} \to \pi^+\pi^-\iota) = (4.0 \pm 0.9) \times 10^{-3}. \quad (28)$$

Analysis of the results presented by OBELIX [7] allows one to determine the quota of K^*K from the all allowed $KK\pi$ modes:

$$\frac{B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to K^*K)}{B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to KK\pi)} = 0.35 \pm 0.04. \quad (29)$$

On the basis of (27)–(29) one can obtain the following important result:

$$\frac{B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to K^*K)}{B(p\bar{p} \to \pi^+\pi^-\iota)} = 0.16 \pm 0.04. \quad (30)$$

It is clear, that with the neglected resonance properties of ι and K^* the left hand size in (30) is the sought-for branching $B(\iota \to K^*K)$. However due to the resonance properties there may be considerable corrections. In order to estimate them let us consider the relations

$$B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to K^*K) = \xi^* B_0(p\bar{p} \to \pi^+\pi^-\iota)\, B, \quad (31)$$

$$B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to \text{“}other\text{”}) = \bar{\xi} B_0(p\bar{p} \to \pi^+\pi^-\iota)\,(1-B). \quad (32)$$

Here the single B is the sought-for branching $B(\iota \to K^*K)$, subscript “0” means that branching $B_0(p\bar{p} \to \pi^+\pi^-\iota)$ is defined in a speculative case of the zero widths of ι and K^*. The “*other*” in (32) means that all other decays of ι are implied, i.e. all decays which occur not via the K^*K. Quantities ξ^* and $\bar{\xi}$ are the factors that guarantee the equality in the relations. Summing up (31) and (32) one gets

$$B(p\bar{p} \to \pi^+\pi^-\iota) = B_0(p\bar{p} \to \pi^+\pi^-\iota)\left[\xi^* B + \bar{\xi}(1-B)\right]. \quad (33)$$

From (31) and (33) one obtains

$$\frac{B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to K^*K)}{B(p\bar{p} \to \pi^+\pi^-\iota)} = \frac{\xi^* B}{\xi^* B + \bar{\xi}(1-B)}. \quad (34)$$

Equating the right hand sizes in (30) and (34), and using the property that ξ^* and $\bar{\xi}$ are the functions on B (see Appendix 1), we obtain the true value of the branching:

$$B(\iota \to K^*K) = 0.40 \pm 0.08. \quad (35)$$

Let us emphasize, that this result is more than twice as large as the naive value in (30). Correction factors ξ^* and $\bar{\xi}$ turn out to be 0.60 ± 0.01 and 2.13 ± 0.03, respectively. (Both they are far from 1, as well.) Multiplying (35) on the total width we come to the final result

$$\Gamma^{\text{exp}}(\iota \to K^*K) = 29.2 \pm 6.1 \text{ MeV}. \quad (36)$$

This result may be compared with the theoretical estimates (24) and (25).

7 Discussion and conclusions

The main results of the present work are the theoretical estimates (24) and (25) for the partial width $\Gamma(\iota \to K^*K)$ — the first one for case when ι is mainly a glueball, and the other one for case when ι is mainly a radial excitation of the $s\bar{s}$ state. (An intermediate case, when ι involves comparable contributions of the glueball and the excited $s\bar{s}$ state is, apparently, not allowed by the data if ι is a single resonance.) Since the above estimates are obtained in χPT approach, their status is close to being model-independent. The assumptions used in deriving the estimates are as follows. First of all, we suppose that ι arises due to the mixing of the glueball, the isoscalar lowest $q\bar{q}$ states (η, η'), and their radial excitations. Then, we identify the excited $n\bar{n}$ state with $\eta(1295)$, and suppose that $K(1460)$ is the radial excitation of the K meson. These assumptions are in agreement with the modern understanding of the 0^{-+} spectrum [2,16] and may be verified by independent methods.

Another important new result is the estimate of $\Gamma(\iota \to K^*K)$ obtained from the combined OBELIX and Crystal Barrel data. The idea to use specifically the OBELIX and Crystall Barrel data is caused by the following reasons. First, these data were collected with the best statistics of ι. Second, and this point is more important, from kinematic reasons the creation of ι with its subsequent decay to $\rho\rho$ and $\omega\omega$ in the $p\bar{p}$ annihilation at rest is strongly suppressed. As a result, one need not take into account these decays while extracting $\Gamma(\iota \to K^*K)$ from the data. This property essentially simplifies the analysis based on the $p\bar{p}$ annihilation data as compared, for example, with the analysis based on the J/ψ radiative decays.

An important technical point of our analysis is that it accurately takes into account the resonance properties of ι and K^*. This point is indeed important since all the decays, considered above, occur near the threshold. We use the relativistic Breit-Wigner amplitude [5] which takes into account the dependence of the partial widths of the resonances on their (varying) invariant masses. As a result, for example, the true value of $B(\iota \to K^*K)$ turns out to be more than twice as large as the "naive" value, which follows directly from the data without taking into account the resonance properties of ι and K^*.

Comparing the theoretical estimate (24) with the experimental estimate (36) we conclude that with the one-resonance structure of ι it may not be a glueball, since the ratio of the theoretical estimate to the experimental one does not exceed 0.28 ± 0.13, which is 5.5σ less than 1. However when ι is mainly a radial excitation of the $s\bar{s}$ state the theoretical estimate agrees with the experimental one. So, taking also into account the results of [4], one may conclude that the $s\bar{s}$ interpretation of ι is possible. However, in accordance with [4], it is only possible when there is noticeable mixing between the $s\bar{s}$ excited state and the higher mass PS glueball (with the mixing angle about 18°). The present study modifies this picture somewhat. Namely, we find that as soon as ι involves a noticeable glueball contribution it must involve also a noticeable ground-state $q\bar{q}$ contribution. This (qualitative) result follows from the fact that the direct $P^S - G$ mixing is suppressed in χPT, but it may be realized indirectly via the $P^S - \phi^0$ and $\phi^0 - G$ mixings. Of course, to describe quantitatively this effect one has to perform a more detailed phenomenological investigation, which might be similar to that of [4] but should take into account the results of the present study.

In conclusion, let us discuss whether our results are applicable to the case when there are two PS resonances in the ι region. Because the two components of ι are most likely the PS glueball and the $s\bar{s}$ excited state that are strongly mixed [24], our estimate (24) is no longer valid in this case since the estimate (23) becomes incorrect. The estimate (25) remains valid, but it may be applied only to the lower ι state. For the upper ι state, the analogous estimate is more than 200 MeV due to increased phase volume. So, both theoretical estimates agree with the experimental ones which in this case may be taken directly from the OBELIX results [7]. (Since in the case of the two-resonance structure of ι the upper ι decays almost only to K^*K, while the lower ι almost does not decay into this channel). So, to specify the nature of the both ι's one needs an additional investigation which must take into account the strong mixing between the lower and upper ι states.

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Appendix 1

This Appendix collects the formulae of calculation of the correction factors ξ, ξ^*, and $\bar{\xi}$.

Let us begin with calculation of ξ which is for the decay $\iota \to K^*K$. Let $\Gamma(\iota \to K^*K; E)$ be the true partial width of ι which mass is equal to E, and $\Gamma_0(\iota \to K^*K; M_\iota)$ be the speculative partial width which is taken with zero width of the K^*. Then we can write

$$\Gamma(\iota \to K^*K; E) = \xi(E)\,\Gamma_0(\iota \to K^*K; M_\iota). \qquad \text{(A1.1)}$$

From (A1.1) there follows

$$\xi(E) = \int_{m_K+m_\pi}^{E-m_K} 2E'dE'\,W(K^*; E') \times \left(\frac{M_{K^*}}{E'}\right)^2 \left[\frac{\mathcal{K}(E; E', m_K)}{\mathcal{K}(M_\iota; M_{K^*}, m_K)}\right]^3. \qquad \text{(A1.2)}$$

Here $\mathcal{K}(M; m1, m2)$ is the module of the three-momentum of particle $m1$ ($m2$) in the rest frame of M in the decay $M \to m1 + m2$. $W(K^*; E)$ is the Breit-Wigner function,

$$W(K^*; E) = \frac{1}{\pi}\,\frac{E\,\Gamma(K^*; E)}{[M_{K^*}^2 - E^2]^2 + [E\,\Gamma(K^*; E)]^2}. \qquad \text{(A1.3)}$$

Let us emphasize, that the correct Breit-Wigner function involves the factor E before the width, and the width must be dependent on the varying invariant mass E of the resonance [5]. In case of the K^* meson the latter dependence of the width is as follows ($r = 0.002$ MeV^{-1})

$$\Gamma(K^*; E) = \Gamma(K^*; M_{K^*})\frac{M_{K^*}^2}{E^2}\left[\frac{\mathcal{K}(E; m_K, m_\pi)}{\mathcal{K}(M_{K^*}; m_K, m_\pi)}\right]^3 \times \frac{1 + [r\mathcal{K}(M_{K^*}; m_K, m_\pi)]^2}{1 + [r\mathcal{K}(E; m_K, m_\pi)]^2}. \qquad \text{(A1.4)}$$

With help of (A1.2)–(A1.4) one can calculate $\xi(E)$. In particular, with $E = 1416$ MeV one obtains $\xi = 1.56$.

The decays $p\bar{p} \to \pi^+\pi^-\iota$ with $\iota \to K^*K$, and $p\bar{p} \to \pi^+\pi^-\iota$ with $\iota \to$ "*other*" may be analysed in the similar way. So, let $\Gamma(\iota; E)$ be the total width of ι which mass is equal to E. Separating K^*K from the "*other*" decay modes, one may represent $\Gamma(\iota; E)$ in the form

$$\Gamma(\iota; E) = \Gamma(\iota \to K^*K; E) + \Gamma(\iota \to \text{"}other\text{"}). \qquad \text{(A1.5)}$$

Here we take into account the property that $\Gamma(\iota \to$ "*other*"$; E)$ is the slowly varying function and neglect its dependence on E.[4] In this approximation

$$\Gamma(\iota \to \text{“}other\text{”}) = (1-B)\,\Gamma(\iota; M_\iota) \tag{A1.6}$$

with $B = B(\iota \to K^*K; M_\iota)$. The rapidly varying function is

$$\Gamma(\iota \to K^*K; E) = \frac{\xi(E)}{\xi(M_\iota)}\, B\Gamma(\iota; M_\iota). \tag{A1.7}$$

In accordance with (A1.5) let us introduce Breit-Wigner functions

$$W(\iota \to K^*K; E) \;=\; \frac{1}{\pi}\,\frac{E\,\Gamma(\iota \to K^*K; E)}{[M_\iota^2 - E^2]^2 + [E\,\Gamma(\iota; E)]^2}, \tag{A1.8}$$

$$W(\iota \to \text{“}other\text{”}; E) \;=\; \frac{1}{\pi}\,\frac{E\,\Gamma(\iota \to \text{“}other\text{”})}{[M_\iota^2 - E^2]^2 + [E\,\Gamma(\iota; E)]^2}. \tag{A1.9}$$

With help of these functions the correction factors ξ^* and $\bar{\xi}$ introduced in (31) and (32) may be represented as

$$\xi^* \;=\; \frac{1}{B}\int_{2m_K+m_\pi}^{M_{p\bar{p}}-2m_\pi} 2EdE\, W(\iota \to K^*K; E)\times \int d\Phi_3(E, m_\pi, m_\pi)/\int d\Phi_3(M_\iota, m_\pi, m_\pi), \tag{A1.10}$$

$$\bar{\xi} = \frac{1}{1-B}\int_{2m_K+m_\pi}^{M_{p\bar{p}}-2m_\pi} 2EdE\, W(\iota \to \text{“}other\text{”}; E)\times \int d\Phi_3(E, m_\pi, m_\pi)/\int d\Phi_3(M_\iota, m_\pi, m_\pi). \tag{A1.11}$$

Here $\int d\Phi_3(E, m_\pi, m_\pi)$ is the phase volume of the annihilation $p\bar{p} \to E\pi\pi$ at rest, which is corrected by the pion derivatives in the decay vertex, see Appendix 3.

Appendix 2

In this Appendix we estimate the contribution of the channel $\iota \to \rho\rho$ to the annihilation of $p\bar{p}$-atom to $\pi\pi\iota$. Namely, we estimate quantity R, where

$$R = \frac{B(p\bar{p} \to \pi^+\pi^-\iota,\, \iota \to \rho\rho)}{B(p\bar{p} \to \pi^+\pi^-\iota)}. \tag{A2.1}$$

At first, let us suppose that the $\rho\rho$ contribution is small, $R \ll 1$. (Then one may use the formulae of Appendix 1.) Putting to use [5], we can write

$$R = \frac{\displaystyle\int_{4m_\pi}^{M_{p\bar{p}}-2m_\pi} 2EdE\, W(\iota \to \rho\rho; E) \int d\Phi_3(E, m_\pi, m_\pi)}{\displaystyle\int_{4m_\pi}^{M_{p\bar{p}}-2m_\pi} 2EdE\, W(\iota; E) \int d\Phi_3(E, m_\pi, m_\pi)}. \tag{A2.2}$$

[4] Indeed, except $\iota \to \rho\rho$ and $\iota \to \omega\omega$, all decays $\iota \to$ "*other*" occur far from the threshold. Concerning $\iota \to \rho\rho$ and $\iota \to \omega\omega$, their contributions are too weak in case of the $p\bar{p}$ annihilation at rest, see Appendix 2

Here $W(\iota; E)$ is the sum of (A1.8) and (A1.9), and the function $W(\iota \to \rho\rho; E)$ is

$$W(\iota \to \rho\rho; E) = \frac{1}{\pi}\,\frac{E\,\Gamma(\iota \to \rho\rho; E)}{[M_\iota^2 - E^2]^2 + [E\,\Gamma(\iota; E)]^2}. \tag{A2.3}$$

The partial width $\Gamma(\iota \to \rho\rho; E)$ is as follows [5]

$$\Gamma(\iota \to \rho\rho; E) = \tfrac{g^2_{\iota\rho\rho}}{8\pi}\int_{2m_\pi}^{E-2m_\pi} 2E'dE'\, W(\rho; E') \times \int_{2m_\pi}^{E-E'} 2E''dE''\, W(\rho; E'')\times [\mathcal{K}(E; E', E'')]^3\, [1 - f(E, E', E'')], \tag{A2.4}$$

where f stands for the interference term, and the function $W(\rho; E)$ is defined analogously to (A1.3), (A1.4).

In order to estimate $g^2_{\iota\rho\rho}/8\pi$ let us take into account the experimental bound $\Gamma(\iota \to \gamma\gamma) \times B(\iota \to K\bar{K}\pi) < 1.2$ kev [25]. Since due to (35) $B(\iota \to K\bar{K}\pi) > 0.4$, from this bound there follows $\Gamma(\iota \to \gamma\gamma) < 3$ kev. Then, putting to use [5] and the VMD model, one can obtain $\Gamma(\iota \to \rho\rho) < 2$ MeV. From this bound, and, again, with help of [5], we obtain $g^2_{\iota\rho\rho}/8\pi < 0.55\,\mathrm{GeV}^{-2}$. This result together with (A2.2)–(A2.4) implies

$$R < 10^{-3}. \tag{A2.5}$$

So, the $\iota \to \rho\rho$ contribution to the annihilation $p\bar{p} \to \pi\pi\iota$ at rest is really negligible. It is clear, that for $\iota \to \omega\omega$ there should be the similar result. Moreover, it should be more strong since the width of the ω is much less than the width of the ρ.

Appendix 3

Since the decay of $p\bar{p}$-atom to $\pi\pi\iota$ occurs near the threshold, it is describable in the framework of χPT. Let us build up the corresponding chiral effective Lagrangian.

With this purpose let us take into account the specific properties of this decay [7]. The first property is that the annihilation $p\bar{p} \to \pi\pi\iota$ at rest is possible only from the isosinglet ${}^1S_0$ state of the $p\bar{p}$-atom. So, its interpolating field, $\mathcal{P}^{\mathcal{N}}$, must transform like P^N. The second property is that the $\pi\pi$ system is produced in S-wave. Therefore, the pion fields may contribute to the Lagrangian either without derivatives or in combinations like $\partial_\mu\,\pi\partial_\mu\pi$. Finally, since the relative angular momentum between the ι and the $\pi\pi$ system is also equal to zero, the interpolating field of ι and that of $\pi\pi$ should contribute without derivatives. The above properties determine the following chiral-invariant effective Lagrangian:

$$L_{\mathcal{P}P\pi\pi} = g_1\,\langle u_\mu u_\mu\{\mathcal{P}, P\}\rangle \;+\; g_2\,\langle[\mathcal{P}, u_\mu][P, u_\mu]\rangle + g_3\,\langle\chi_+\{\mathcal{P}, P\}\rangle \;+\; g_4\,\langle\chi_-\mathcal{P}\rangle + O(p^4). \tag{A3.1}$$

Here $\mathcal{P}$ stands for the baryon-antibaryon atom, P stands for the nonet of the excited $q\bar{q}$ states and ϕ^0. (The glueball interpolating field does not contribute to the Lagrangian due to the reasons discussed in Sects. 2 and 4. Note, the

latter property does not affect the final result.) In what follows we consider $\mathcal{P} = \mathcal{P}^{\mathcal{N}} \lambda^N/2$.

Putting $m_\pi^2 = 0$, one can show that only ϕ^0 contributes to $p\bar{p} \to \pi\pi\iota$. (Generally speaking, the excited state P^N can contribute, too, but it does not contribute to ι.) After the superfluous terms are rejected, in right hand size in (A3.1) there remain

$$\frac{g_1}{2F^2} \mathcal{P}^{\mathcal{N}} \phi^0 \left(\partial_\mu \pi^0 \partial_\mu \pi^0 + 2\, \partial_\mu \pi^+ \partial_\mu \pi^- \right). \qquad \text{(A3.2)}$$

From (A3.2) there follows the sought-for result

$$L_{\mathcal{N}\iota\pi\pi} \propto \mathcal{P}^{\mathcal{N}} P^{\iota} \left(\partial_\mu \pi^0 \partial_\mu \pi^0 + 2\, \partial_\mu \pi^+ \partial_\mu \pi^- \right). \qquad \text{(A3.3)}$$

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